Problem Set 3

Due: Sept. 17, 2008

1. In this problem, you will derive the Gell-Mann-Okubo mass formula for the octet baryons. You will need the transformation law for a 3 of $SU(3)$:

$$q_i \to U_{ij} q_j, \qquad \overline{q}_i \to \overline{q}_j (U^\dagger)_{ji}$$

where $U \in SU(3)$ and i, j are $SU(3)$ flavor indices.

(a) An octet of $SU(3)$ comes from $3 \otimes \overline{3} = 8 \oplus 1$, and two come from $3 \otimes 3 \otimes 3$. An antiquark and a specific combination of two quarks both transform in the $\overline{3}$ of $SU(3)$. Specifically,

$$\overline{q}_i \quad \text{and} \quad \epsilon_{ijk} q_j q_k$$

transform both as $\overline{3}$'s in $SU(3)$. Prove that these two objects transform the same.

Solution:

We can write

$$\epsilon_{ijk} q_j q_k \quad \to \quad \epsilon_{ijk} q'_j q'_k = \epsilon_{ijk} U_{jj'} U_{kk'} q_{j'} q_{k'}$$

But we know that

$$\epsilon_{ijk} U_{ii'} U_{jj'} U_{kk'} = \epsilon_{i'j'k'}$$

because the completely antisymmetric tensor for any group is invariant under a group transformation. We can multiply both sides by $U^\dagger_{i'm}$ to get

$$\epsilon_{mjk} U_{jj'} U_{kk'} = \epsilon_{i'j'k'} U^\dagger_{i'm}$$

so

$$\begin{aligned} \epsilon_{ijk} q'_j q'_k &= \epsilon_{i'j'k'} U^\dagger_{i'i} q_{j'} q_{k'} \\ &= U^\dagger_{i'i} (\epsilon_{i'j'k'} q_{j'} q_{k'}) \end{aligned}$$

and since

$$\overline{q}_i \to \overline{q}_{i'} U^\dagger_{i'i}$$

these two objects transform the same.

(b) We can write a "nonet" combination of the quarks and antiquarks as

$$B_{ij} = q_i \overline{q}_j$$

where $i, j = 1, 2, 3$, and so B is a 3×3 matrix. B is a reducible representation of $3 \otimes \overline{3}$, as we know. Show how

$$\mathcal{B}_{ij} = B_{ij} - \frac{1}{3} \operatorname{Tr}(B) \delta_{ij}, \quad \text{and} \quad \operatorname{Tr}(B)$$

each transform under $SU(3)$ rotations. Note that we could now replace $\overline{q}_j$ with $\epsilon_{jik} q_i q_k$ to get the same transformation properties, and this would correspond to a baryon physically.

Solution:

So under an $SU(3)$ transformation,

$$\operatorname{Tr}(B) = B_{ii} \to U_{ii'} B_{i'j'} U^\dagger_{j'i}$$

but

$$U_{ii'}U^\dagger_{j'i} = \delta_{i'j'}$$

so

$$\mathrm{Tr}(B) \to B_{i'i'} = \mathrm{Tr}(B)$$

is a singlet under $SU(3)$.

As for $\mathcal{B}$:

$$\begin{aligned}
\mathcal{B}_{ij} &= B_{ij} - \frac{1}{3}\,\mathrm{Tr}(B)\delta_{ij} \\
&\to U_{ii'}B_{i'j'}U^\dagger_{j'j} - \frac{1}{3}\,\mathrm{Tr}(B)\delta_{ij} \\
&= U_{ii'}B_{i'j'}U^\dagger_{j'j} - \frac{1}{3}\,\mathrm{Tr}(B)\delta_{i'j'}U_{ii'}U^\dagger_{j'j} \\
&= U_{ii'}\left[B_{i'j'} - \frac{1}{3}\,\mathrm{Tr}(B)\delta_{i'j'}\right]U^\dagger_{j'j} \\
&= U_{ii'}\mathcal{B}_{i'j'}U^\dagger_{j'j}
\end{aligned}$$

So this transforms as an octet under $SU(3)$.

(c) Gell-Mann's assumption was that the strong interactions would contribute two terms to the masses of the baryons with the Hamiltonian:

$$H_{\text{strong}} = H_{\overline{m}} + H_{\delta m}$$

The first term would contribute the average mass, M_0, of a given multiplet (*ie*, if $SU(3)$ were exact we would just have

$$M_p = M_n = M_\Sigma = M_\Xi = M_\Lambda = M_0\ .$$

The second term splits the masses in a given multiplet. Since this is due to strong interactions it should be independent of charge and isospin (assuming that isospin is still a good symmetry), and thus must involve the $T^8 = \lambda^8/2$ generator of the octet, which is diagonal in isospin space.

The two combinations

$$\mathrm{Tr}(\mathcal{B}^\dagger\mathcal{B}T^8),\ \text{and}\ \mathrm{Tr}(\mathcal{B}^\dagger T^8\mathcal{B})$$

are the only terms that break $SU(3)$ and are invariant under $SU(2)$ involving $\mathcal{B}$, so

$$H_{\delta m} = \delta m_1\,\mathrm{Tr}(\mathcal{B}^\dagger\mathcal{B}T^8) + \delta m_2\,\mathrm{Tr}(\mathcal{B}^\dagger T^8\mathcal{B})$$

with two unknown parameters, $\delta m_{1,2}$.

We will use the following octet representation of the baryons:

$$\mathcal{B} = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

where all elements are complex operators, so $\mathcal{B}^\dagger$ contains $\Lambda^\dagger, p^\dagger$, etc., and the fact that a mass term for baryons are of the form

$$M_p p^\dagger p + M_n n^\dagger n + M_\Sigma \Sigma^\dagger\Sigma + \cdots$$

for baryons, we would like to determine the masses of the octet baryons. Evaluate the traces in $H_{\delta m}$ in terms of these baryons.

Solution:

$$\mathrm{Tr}(\mathcal{B}^\dagger \mathcal{B} T^8) = \frac{1}{2\sqrt{3}}\left[-2n^\dagger n - 2p^\dagger p - \Lambda^\dagger\Lambda + (\Xi^0)^\dagger\Xi^0 + (\Xi^-)^\dagger\Xi^- + (\Sigma^0)^\dagger\Sigma^0 + (\Sigma^+)^\dagger\Sigma^+ + (\Sigma^-)^\dagger\Sigma^-\right]$$

$$\mathrm{Tr}(\mathcal{B}^\dagger \mathcal{B} T^8) = \frac{1}{2\sqrt{3}}\left[n^\dagger n + p^\dagger p - \Lambda^\dagger\Lambda - 2(\Xi^0)^\dagger\Xi^0 - 2(\Xi^-)^\dagger\Xi^- + (\Sigma^0)^\dagger\Sigma^0 + (\Sigma^+)^\dagger\Sigma^+ + (\Sigma^-)^\dagger\Sigma^-\right]$$

(d) For each baryon, add in the contribution M_0 from $H_{\overline{m}}$ (common to all baryons), and determine the masses of the baryons in terms of $M_0, \delta m_1$, and δm_2. With isospin a good symmetry, we have

$$m_n = m_p \equiv m_N, \qquad m_{\Sigma^+} = m_{\Sigma^0} = m_{\Sigma^-} \equiv m_\Sigma, \qquad m_{\Xi^-} = m_{\Xi^0} \equiv m_\Xi$$

Solution:

We can already see, by construction, that all the baryons with the same isospin will have the same masses (this is how we constructed the Hamiltonian). We get

$$\begin{aligned} m_N &= M_0 - \frac{\delta m_1}{\sqrt{3}} + \frac{\delta m_2}{2\sqrt{3}} \\ m_\Sigma &= M_0 + \frac{\delta m_1}{2\sqrt{3}} + \frac{\delta m_2}{2\sqrt{3}} \\ m_\Xi &= M_0 + \frac{\delta m_1}{2\sqrt{3}} - \frac{\delta m_2}{\sqrt{3}} \\ m_\Lambda &= M_0 - \frac{\delta m_1}{2\sqrt{3}} - \frac{\delta m_2}{2\sqrt{3}} \end{aligned}$$

(e) Finally, use these equations to derive the Gell-Mann-Okubo relation

$$m_\Lambda = \frac{1}{3}\left[2(m_N + m_\Xi) - m_\Sigma\right]$$

Solution:

First

$$2(m_N + m_\Xi) = 4M_0 - \frac{\delta m_1}{\sqrt{3}} - \frac{\delta m_2}{\sqrt{3}}$$

$$2(m_N + m_\Xi) - m_\Sigma = 3M_0 - 3\frac{\delta m_1}{2\sqrt{3}} - 3\frac{\delta m_2}{2\sqrt{3}}$$

And the right-hand side is precisely $3m_\Lambda$.

2. Determine, using the quark model, the magnetic moments in units of the nuclear magneton, of all the members of the decuplet. Use whatever method you need to fix the unknown parameters (the constituent quark masses, m_u, m_d, m_s), for example you can use the quark model predictions for three of the octet baryons to set those, and then predict the decuplet. For those which have been measured, compare with the experimental number.

Solution:

First we need all the states of the decuplet, which we will put all in the $+3/2$ spin state, because that is how the magnetic moment is defined, so we will not denote the spin state explicitly. Thus, all quarks in these states are spin-up.

$$|\Delta^{++}\rangle = |uuu\rangle \qquad |\Delta^+\rangle = \frac{1}{\sqrt{3}}\left(|uud\rangle + |udu\rangle + |duu\rangle\right)$$

$$|\Delta^0\rangle = \frac{1}{\sqrt{3}}\left(|udd\rangle + |dud\rangle + |ddu\rangle\right) \qquad |\Delta^-\rangle = |ddd\rangle$$

$$
\begin{aligned}
|\Sigma^+\rangle &= \frac{1}{\sqrt{3}}\left(|uus\rangle + |usu\rangle + |suu\rangle\right) \\
|\Sigma^0\rangle &= \frac{1}{\sqrt{6}}\left(|usd\rangle + |uds\rangle + |dus\rangle + |dsu\rangle + |sud\rangle + |sdu\rangle\right) \\
|\Sigma^-\rangle &= \frac{1}{\sqrt{3}}\left(|dds\rangle + |dsd\rangle + |sdd\rangle\right)
\end{aligned}
$$

$$
\begin{aligned}
|\Xi^0\rangle &= \frac{1}{\sqrt{3}}\left(|uss\rangle + |sus\rangle + |ssu\rangle\right) \\
|\Xi^-\rangle &= \frac{1}{\sqrt{3}}\left(|dss\rangle + |sds\rangle + |ssd\rangle\right)
\end{aligned}
$$

and

$$|\Omega^-\rangle = |sss\rangle$$

Using the magnetic moment operator

$$\mu_i = +Q_i \frac{e}{2m_i}$$

where the plus sign is used here because all the spins are up, and $Q_i = 2/3$ for $i = u$ and $-1/3$ for $i = d, s$. Also we recall the states are all orthonormal, so we have for example

$$
\begin{aligned}
\mu_{\Delta^{++}} &= \langle \Delta^{++}|\mu|\Delta^{++}\rangle \\
&= \langle uuu|\mu|uuu\rangle \\
&= (1+1+1)\frac{e}{2m_u} \\
&= 3\mu_u
\end{aligned}
$$

Similarly:

$$
\begin{aligned}
\mu_{\Delta^+} &= \frac{1}{3}3\left(2\mu_u + \mu_d\right) = \left(2\mu_u + \mu_d\right) \\
\mu_{\Delta^0} &= \frac{1}{3}3\left(\mu_u + 2\mu_d\right) = \left(\mu_u + 2\mu_d\right) \\
\mu_{\Delta^-} &= 3\mu_d \\
\mu_{\Sigma^+} &= \left(2\mu_u + \mu_s\right) \\
\mu_{\Sigma^0} &= \left(\mu_u + \mu_d + \mu_s\right) \\
\mu_{\Sigma^-} &= \left(\mu_d + \mu_s\right) \\
\mu_{\Xi^0} &= \left(\mu_u + 2\mu_s\right) \\
\mu_{\Xi^-} &= \left(\mu_d + 2\mu_s\right) \\
\mu_{\Omega^-} &= 3\mu_s
\end{aligned}
$$

Using the expressions in class (where the Σ^+ below is the octet Σ^+). If we again assume isospin symmetry, so $m_d = m_u$ and thus $\mu_u = -2\mu_d$, these expressions become

$$
\begin{aligned}
\mu_p &= \mu_u \\
\mu_n &= -\frac{1}{2}\mu_u \\
\mu_{\Sigma^+} &= \frac{1}{3}\left(4\mu_u - \mu_s\right)
\end{aligned}
$$

and experimentally:

$$
\begin{aligned}
\mu_p &= 2.793\mu_N \\
\mu_n &= -1.913\mu_N \\
\mu_{\Sigma^+} &= 2.46\mu_N
\end{aligned}
$$

At this point we realize we have already made some error, because of the fact that we know the proton and neutron magnetic moments are not quite related by this ratio. The easiest thing to do is to take say, the Ω^- mass, which is 1672 MeV, and assume the constituent strange quark mass is one-third of this, to get

$$m_s \approx 550\text{MeV} \approx \frac{m_N}{1.68}$$

This is a more accurate assumption in some sense than we did in class. This means

$$\mu_s = -\frac{1}{3}1.68\mu_N = -0.56\mu_N$$

and recall

$$\mu_u = 2\mu_N, \qquad \mu_d = -\mu_N$$

Putting these into the decuplets, we get:

$$\begin{aligned}
\mu_{\Delta^{++}} &= 6\mu_N \\
\mu_{\Delta^{+}} &= 3\mu_N \\
\mu_{\Delta^{0}} &= 0 \\
\mu_{\Delta^{-}} &= -3\mu_N \\
\mu_{\Sigma^{+}} &= 3.44\mu_N \\
\mu_{\Sigma^{0}} &= 0.44\mu_N \\
\mu_{\Sigma^{-}} &= -1.56\mu_N \\
\mu_{\Xi^{0}} &= 0.88\mu_N \\
\mu_{\Xi^{-}} &= -2.12\mu_N \\
\mu_{\Omega^{-}} &= -1.68\mu_N
\end{aligned}$$

Of these, the measured values are:

$$\begin{aligned}
\mu_{\Delta^{++}} &= (3.7 - 7.5)\mu_N \\
\mu_{\Delta^{+}} &= (2.7 \pm 1.0 \pm 1.3 \pm 1.5 \pm 3)\mu_N \\
\mu_{\Omega^{-}} &= -(2.02 \pm 0.05)\mu_N
\end{aligned}$$

So our numbers are not far off. If we wanted to determine μ_s there is a problem in using the Σ^+ number, as it puts the up and down quarks on different footings. A safer method would be to use the Λ magnetic moment, which in the quark model is

$$\mu_\Lambda = \mu_s$$

and this gives

$$\mu_s = -0.613\mu_N$$

which would be in rough agreement with what we have above. Notice that there are significant differences in predictions depending on the method used to determine these "quark magnetic moments."

3. What about baryons with heavy quarks, the "charmed baryons" and "beautiful baryons"?

   (a) Show that there are six spin-1/2 baryons containing a charm quark and two light quarks (u, d, s) that transform in the 6 of $SU(3)$, with flavor content

   $$\Sigma_c^{++} \sim uuc\ , \quad \Sigma_c^{+} \sim udc\ , \quad \Sigma_c^{0} \sim ddc\ , \quad \Xi_c^{\prime +} \sim usc\ , \quad \Xi_c^{\prime 0} \sim dsc\ , \quad \Omega_c^{0} \sim ssc$$

   and 3 baryons

   $$\Xi_c^{+} \sim usc\ , \quad \Xi_c^{0} \sim dsc\ , \quad \Lambda_c^{+} \sim udc$$

   which transform in the $\overline{3}$ of $SU(3)$. To do this, work out the different combinations that give a spin-1/2 baryon and use the fact that the total state must be completely symmetric in spin and flavor, since it is to combine with a completely antisymmetric state in color. Be sure to write down the full states, with the correct normalization, and describe the correct symmetry properties.

Solution:

We know that there are 9 possibilities, coming from the different possibilities of putting 3 light quarks into two slots of the charmed baryons. We can use the fact that the charm quark would be a singlet under $SU(3)$, so we only have to worry about the light quark combinations. First let's look at the different irreducible representations under the $SU(2)$ isospin subgroup. Here we have terms which are symmetric under the interchange of the u, d quarks, and these are isospin-1 states:

$$uu \ , \quad \frac{1}{\sqrt{2}}(ud + du) \ , \quad dd$$

and then an antisymmetric combination, with isospin-0:

$$\frac{1}{\sqrt{2}}(ud - du)$$

Then, we take the first set of combinations, and interchange the u, d quarks with s quarks, to get two combinations

$$\frac{1}{\sqrt{2}}(us + su) \ , \quad \frac{1}{\sqrt{2}}(ds + sd)$$

and again to get

$$ss$$

So all of these symmetric combinations form a sextet under $SU(3)$ (we know they must belong to a single representation since they have the same symmetry properties). Taking the antisymmetric combination and interchanging u, d quarks with s, we have two more combos:

$$\frac{1}{\sqrt{2}}(us - su) \ , \quad \frac{1}{\sqrt{2}}(sd - ds)$$

and this is a triplet. We know it is a $\bar{3}$, because we can write each element as

$$\epsilon_{ijk} q_j q_k$$

and this is a $\bar{3}$, as shown in the first problem.

Now, to form the states, we want to form spin-1/2 states from these. Looking at the symmetric states under isospin, we can start with (all are in the spin-up state)

$$|\Sigma_c^{++}\rangle \quad \sim \quad |uuc\rangle \left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right)$$

This is symmetric under the interchange under the first two quarks, and now we iterate and normalize:

$$\begin{aligned} |\Sigma_c^{++}\rangle \quad &= \quad \frac{1}{\sqrt{18}}\Big[|uuc\rangle \left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right) + |ucu\rangle \left(|\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\rangle - 2|\uparrow\downarrow\uparrow\rangle\right) \\ &\quad + |cuu\rangle \left(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle - 2|\downarrow\uparrow\uparrow\rangle\right)\Big] \\ &\equiv \quad \frac{1}{\sqrt{18}}\left[|uuc\rangle\chi_{\uparrow\uparrow\downarrow} + |ucu\rangle\chi_{\uparrow\downarrow\uparrow} + |cuu\rangle\chi_{\downarrow\uparrow\uparrow}\right] \end{aligned}$$

and running down the rest:

$$
\begin{aligned}
|\Sigma_c^+\rangle &= \frac{1}{6}\left[(|udc\rangle + |duc\rangle)\,\chi_{\uparrow\uparrow\downarrow} + (|dcu\rangle + |ucd\rangle)\,\chi_{\uparrow\downarrow\uparrow} + (|cud\rangle + |cdu\rangle)\,\chi_{\downarrow\uparrow\uparrow}\right] \\
|\Sigma_c^0\rangle &= \frac{1}{\sqrt{18}}\left[|ddc\rangle\chi_{\uparrow\uparrow\downarrow} + |dcd\rangle\chi_{\uparrow\downarrow\uparrow} + |cdd\rangle\chi_{\downarrow\uparrow\uparrow}\right] \\
|\Xi_c^{\prime +}\rangle &= \frac{1}{6}\left[(|usc\rangle + |suc\rangle)\,\chi_{\uparrow\uparrow\downarrow} + (|scu\rangle + |ucs\rangle)\,\chi_{\uparrow\downarrow\uparrow} + (|cus\rangle + |csu\rangle)\,\chi_{\downarrow\uparrow\uparrow}\right] \\
|\Xi_c^{\prime 0}\rangle &= \frac{1}{6}\left[(|sdc\rangle + |dsc\rangle)\,\chi_{\uparrow\uparrow\downarrow} + (|dcs\rangle + |scd\rangle)\,\chi_{\uparrow\downarrow\uparrow} + (|csd\rangle + |cds\rangle)\,\chi_{\downarrow\uparrow\uparrow}\right] \\
|\Omega_c^0\rangle &= \frac{1}{\sqrt{18}}\left[|ssc\rangle\chi_{\uparrow\uparrow\downarrow} + |scs\rangle\chi_{\uparrow\downarrow\uparrow} + |css\rangle\chi_{\downarrow\uparrow\uparrow}\right]
\end{aligned}
$$

For the $\bar{3}$, we do the same thing but start with terms that are antisymmetric under the isospin, to get

$$
\begin{aligned}
|\Xi_c^+\rangle &= \frac{1}{6}\left[(|usc\rangle - |suc\rangle)\,\chi'_{\uparrow\uparrow\downarrow} + (|scu\rangle - |ucs\rangle)\,\chi'_{\uparrow\downarrow\uparrow} + (|cus\rangle - |csu\rangle)\,\chi'_{\downarrow\uparrow\uparrow}\right] \\
|\Xi_c^0\rangle &= \frac{1}{6}\left[(|sdc\rangle - |dsc\rangle)\,\chi'_{\uparrow\uparrow\downarrow} + (|dcs\rangle - |scd\rangle)\,\chi'_{\uparrow\downarrow\uparrow} + (|csd\rangle - |cds\rangle)\,\chi'_{\downarrow\uparrow\uparrow}\right] \\
|\Lambda_c^+\rangle &= \frac{1}{6}\left[(|udc\rangle - |duc\rangle)\,\chi'_{\uparrow\uparrow\downarrow} + (|dcu\rangle - |ucd\rangle)\,\chi'_{\uparrow\downarrow\uparrow} + (|cud\rangle - |cdu\rangle)\,\chi'_{\downarrow\uparrow\uparrow}\right]
\end{aligned}
$$

where

$$\chi'_{\uparrow\uparrow\downarrow} \equiv (|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle)$$

(b) Assuming (naively) the masses of baryons can be written as

$$m_B = m_1 + m_2 + m_3$$

where m_i are the "constituent quark masses," then we can estimate the masses of these charmed baryons. Given that the mass of the proton is roughly 1 GeV, we can approximate $m_u \approx m_d \approx 330$ MeV, and since the $\Omega^- \sim sss$ mass is roughly 1.7 GeV, $m_s \approx 570$ MeV. Use this to predict the masses of the above states. For this, you can assume the constituent charm quark mass is just its mass, 1.3 GeV.

(c) Look up in the particle data book the values of the masses of the above baryons (given that we are defining the parities to be positive, so these are all $1/2^+$ states), and compare those values to our naïve quark model estimates. [Note: You obviously won't be able to distinguish with this naïve estimate the differences between the sextet and anti-triplet.]

Solution:

This problem is using just a naive assumption, so we really only care about the quark content, and the u, d masses are equal, so we have no difference between the Ξ_c and Ξ'_c baryons, and we get (experimental numbers are in parentheses):

$$
\begin{aligned}
m_{\Sigma_c} &= 2m_u + m_c = 2.36\text{GeV}(2.455) \\
m_{\Xi_c} &= m_u + m_s + m_c = 2.6\text{GeV}(2.468) \\
m_{\Omega_c^0} &= 2m_s + m_c = 2.84\text{GeV}(2.698) \\
m_{\Lambda_c} &= 2m_u + m_c = 2.36\text{GeV}(2.286)
\end{aligned}
$$

So these all agree to roughly 5%, which is not so bad.

(d) Do the same as problem 3b but with $c \to b$, and $m_b = 5$ GeV. The only one of these baryons discovered is the $\Lambda_b^0 \sim udb$ (note the charge is different because the electric charge of the bottom quark is $-1/3e$), and it has a mass of $m_{\Lambda_b^0} = 5620.2$ MeV. Why should the quark model work so much better here? [What assumption did we implicitly use here to describe the baryon in terms of constituent quark masses?]

Solution:

The states are the same, with $c \to b$, and we have

$$\begin{aligned} m_{\Sigma_b} &= 2m_u + m_c = 5.66\text{GeV}(---) \\ m_{\Xi_b} &= m_u + m_s + m_c = 5.9\text{GeV}(---) \\ m_{\Omega_b^0} &= 2m_s + m_c = 6.14\text{GeV}(---) \\ m_{\Lambda_b} &= 2m_u + m_c = 5.66\text{GeV}(5.6202) \end{aligned}$$

This agrees to 0.7%! This makes sense, because the quark model (and specifically this method of estimating baryon masses) effectively takes the non-relativistic limit. The larger the quark mass, the more accurate this approximation is.

Additional Problem (not to be turned in)

1. When simulating fermions on a spacetime lattice, there is a duplication of quark species, so that for every quark flavor, you have 4 *tastes* (this is the so-called doubling problem). Usually one simulates three flavors, u, d, s, each with four tastes and the relevant symmetry group then is $SU(12)$.

   (a) Using Young Tableaux, work out the number of mesons we would expect to see, and also the different dimensions of the baryons that would come out. In other words, work out $12 \otimes \overline{12}$ and $12 \otimes 12 \otimes 12$.

   (b) Of the resulting irreducible representations that arise for baryons, which ones would correspond to "physical" multiplets [You can just make an argument here, no mathematical proof is needed.]?

   (c) Now do the same with say, the charmed baryons. For heavy quarks, we find no doubling to occur (because of the heavy mass of the quark), so while the light quarks transform under $SU(12)$, the heavy quarks are $SU(12)$ singlets. What are the resulting charmed baryon states with a single charm quark (*ie*, what are the dimensions of the possible irreducible representations?)?