Problem Set 5

Due: Oct. 20, 2007

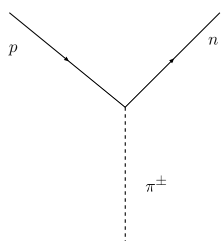
Problems 1-3 concern an effective theory for pions and nucleons, where m_π is the common mass of all three pions and M is the mass of the nucleons. The interaction Lagrangian is given by

$$\mathcal{L}_I = -ig_A \left[\sqrt{2}\, \overline{\psi}_p \gamma_5 \psi_n \pi^+ + \sqrt{2}\, \overline{\psi}_n \gamma_5 \psi_p \pi^- + \left(\overline{\psi}_p \gamma_5 \psi_p - \overline{\psi}_n \gamma_5 \psi_n \right) \pi^0 \right]$$

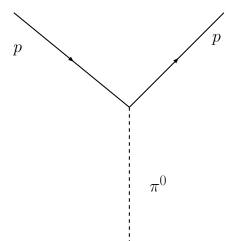
and there is only one coupling, restricted by isospin invariance.

1. For $\mathcal{L}_I$ to be Lorentz invariant, how must the pions transform under Lorentz transformations (scalar, pseudoscalar, vector, axial-vector, or tensor)?

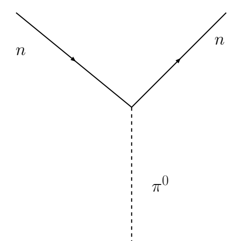
2. The interaction vertices are given by



$= \sqrt{2} g \gamma_5$,



$= g\gamma_5$,



$= -g\gamma_5$.

   There are 10 possible pion-nucleon scattering events:

   (a) $\pi^+ + p \to \pi^+ + p$
   (b) $\pi^0 + p \to \pi^0 + p$
   (c) $\pi^- + p \to \pi^- + p$
   (d) $\pi^+ + n \to \pi^+ + n$
   (e) $\pi^0 + n \to \pi^0 + n$
   (f) $\pi^- + n \to \pi^- + n$
   (g) $\pi^+ + n \to \pi^0 + p$
   (h) $\pi^0 + n \to \pi^- + p$
   (i) $\pi^- + p \to \pi^0 + n$
   (j) $\pi^0 + p \to \pi^+ + n$

   Calculate the amplitudes for these processes, and use isospin to relate them to just two amplitudes: One with isospin 1/2 and one with isospin 3/2: $\mathcal{M}_{1/2}, \mathcal{M}_{3/2}$.

3. Calculate the total cross sections for elastic $\pi^+ p$ and $\pi^- n$ scattering in the center of mass frame. These cross sections do not hold near $E_{CM} \approx 1232$ MeV. Explain.

4. Show that the scattering amplitude and the total cross section are related by the identity

$$\sigma = \frac{4\pi}{k} \mathrm{Im} f_{\mathbf{k}}(0)$$

   which is known as the *Optical Theorem*.

5. In connecting the relativistic normalization used in field theory, the first Born approximation reads

$$\langle p'|iT|p\rangle = -i\tilde{V}(\mathbf{p}' - \mathbf{p})(2\pi)\delta(E_{\mathbf{p}'} - E_{\mathbf{p}})$$

where $\tilde{V}$ is the Fourier transform of the potential, and T is the Transition matrix which we relate to the amplitude. Show that in the non-relativistic limit, $e^+e^- \to e^+e^-$ scattering in QED does indeed give the Coulomb potential, while in the Yukawa theory (replacing the photon with a massive spin-0 boson), you get

$$V(r) \propto \frac{e^{-mr}}{r}$$

Also show that the Yukawa theory is *always* attractive, while for QED it depends on the sign of the fermions. [Note, this is why Yukawa proposed this for the strong interactions (see the pion-nucleon theory above), as it ensured a purely attractive theory for the nucleons.]