Problem Set 6 ("Midterm")

Due: By classtime, Oct. 27, 2008, **no exceptions**

For this assignment, you are not allowed to work together, although you can use any notes and texts. Additionally, if you have any questions on the problems, email me (as I will be out of town this is your only option), and I will respond to all students with my answer. Overall this assignment will be part of your homework grade, but is worth **100 points**.

1. [10 pts] Derive the full vertex in terms of the electromagnetic form factor, $G_\pi(q^2)$ for the π^+, recalling that the pions are spin-zero.

2. [10 pts] Show using charge conjugation invariance that the electromagnetic form factor for the π^0 is zero (use the same result from problem 1 to get the form, and then show that $G_\pi = 0$ for all q^2 with charge conjugation invariance). Why does this not hold for the neutron?

3. [30 pts] Now derive the electromagnetic form factors for the ρ^+ meson. The full vertex will have the form
$$\epsilon^*_\alpha(p')\left(e\Gamma^{\alpha\mu\beta}\right)\epsilon_\beta(p)$$
where the polarization tensors of the ρ satisfy
$$p^{'\alpha}\epsilon_\alpha(p') = p^\beta\epsilon_\beta(p) = 0$$
and
$$p^2 = (p')^2 = m_\rho^2$$
Be sure to write down *all possible* terms that arise due to Lorentz invariance, and use charge conservation ($(p'-p)_\mu\Gamma^{\alpha\mu\beta} = 0$) to reduce the number of form factors to three. [In general, for a particle of spin-s, there are $2s+1$ electromagnetic form-factors which in the $q^2 \to 0$ limit, lead to $2s+1$ electromagnetic moments. In the case of a spin-one field, you get in addition to the electric charge and magnetic dipole moment, the electric quadrupole moment.]

4. [20 pts] Calculate the differential cross-section in terms of the form factors for $e^-\rho^+ \to e^-\rho^+$ scattering. For this, you can average/sum over initial/final polarization states of the ρ, and so you need
$$\sum_i \epsilon_\mu^{(i)}(p)\epsilon_\nu^{*(i)}(p) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_\rho^2}$$
where i runs over the *three* spin states of the ρ.

5. [30 pts] Now calculate the decay rate for the process $B \to De\bar{\nu}_e$, where B, D are heavy-light spin-zero mesons with quark content $D \sim c\bar{q}, B \sim b\bar{q}$, and $q = u, d, s$. First, we can express the matrix element in terms of two form factors:
$$\langle D(p')|V^\mu|B(P)\rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu$$
Neglecting the electron mass, we can ignore f_-. The matrix element for this process is given by
$$\mathcal{M}(B \to De\bar{n}u_e) = \sqrt{2}G_F V_{cb} f_+(q^2)(p+p')^\mu \bar{u}(p_e)\gamma_\mu\frac{1-\gamma_5}{2}v(p_{\nu_e})$$
with G_F the Fermi coupling constant (relevant to the weak interactions), and V_{cb} is a CKM matrix element which describes the likelihood of the $b \to c$ transition. Calculate the decay rate using this, summing over the final spins. To keep the q^2-dependence explicit, calculate $d\Gamma/dq^2$, where
$$\frac{d\Gamma}{dq^2} = d\Gamma\delta[q^2 - (p'-p)^2]$$
The tricky part of this is to calculate the 3-body phase space integrals.