Problem Set 8

Due: By Dec. 19, 2008 **(no exceptions!)**

We haven't discussed adding nucleons to chiral perturbation theory in class, but it is, for the most part, a straightforward process.

1. We define the nucleon field as before
$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$
and under the $SU(2)$ group of isospin transformations (which is the vector subgroup of the $SU(2)_L \times SU(2)_R$ chiral symmetry), $N \to VN$, with $V \in SU(2)$. Just as in the case of the heavy-lights, we can define our transformation law under the chiral symmetry as
$$N \to \mathbb{U}(x)N$$
where $\mathbb{U}$ is the same $SU(2)$ matrix which arises in the transformation law of the ξ field we defined in class. Using ξ (or more conveniently, $\mathbb{V}_\mu, \mathbb{A}_\mu$), construct the terms nucleon chiral Lagrangian. Note, without chiral symmetry, we would just have the Lagrangian
$$\mathcal{L}_{\text{nucleons}} = \overline{N}(i\partial\!\!\!/ - M_0)N$$
where M_0 is the leading-order mass of the nucleon, and this term (as in the case of the heavy-lights) is *not* invariant under the chiral transformation law.

2. Expand the ξ fields to linear order in the pion (note, we're discussing $SU(2)$ here, so use the appropriate version of ξ), and ultimately write the $\mathbb{A}_\mu$ term with $\pi^\pm, \pi^0$. There should be one term with π^+, one with π^-, and two with π^0. Show that you obtain precisely the Lagrangian from Problem Set 5 to this order.

3. On a different note, the rate of decay for $D^{*+} \to D\pi^+$ is
$$\Gamma = \frac{g_\pi^2 |\mathbf{p}_\pi|^2}{12\pi f^2}$$
What would it be for $D^{*0} \to D^0\pi^0$? Think of the quark assignments
$$D^{*+} \sim c\bar{d}\ , \quad D^{*0} \sim c\bar{u}\ , \quad D^+ \sim c\bar{d}\ , \quad D^0 \sim c\bar{u}\ , \quad \pi^+ \sim u\bar{d}\ , \quad \pi^0 \sim u\bar{u} + d\bar{d}$$
(It is a simple exercise to work out the decay rate yourself from the Heavy-Light Lagrangian, so I urge you to do it.)