Evaluating the Bubble diagram

Evaluate the bubble diagram in SChPT, $B(t)$ in the fully degenerate limit, which is the Fourier transform of

$$B(p) = \mu^2 \frac{1}{L^3} \int \frac{dk_4}{2\pi} \sum_{\mathbf{k}} \Bigg\{ -4 \Bigg[\frac{1}{3} \frac{1}{(k+p)^2 + M^2_{\pi^0_I}} \frac{1}{(k^2 + M^2_{\pi_I})} + a^2 \delta'_V \frac{1}{(k+p)^2 + M^2_{\pi^0_V}} \frac{1}{(k^2 + M^2_{\pi^0_V})(k^2 + M^2_{\eta'_V})}$$
$$+ a^2 \delta'_A \frac{1}{(k+p)^2 + M^2_{\pi^0_A}} \frac{1}{(k^2 + M^2_{\pi^0_A})(k^2 + M^2_{\eta'_A})} \Bigg]$$
$$+ C \frac{1}{4} \sum_{b=1}^{16} \Bigg[3 \frac{1}{(k+p)^2 + M^2_{\pi^0_b}} \; \frac{1}{k^2 + M^2_{\pi^0_b}} \Bigg] \Bigg\} ,$$

with $\mathbf{p} = 0$, L is the spatial extent, and $C = 1(1/4)$ for unrooted (rooted) staggered quarks. First, assume the spatial $\mathbf{k}$ are finite, and the time direction is infinite, so

$$B(t) = \frac{1}{L^3} \sum_{p_1, p_2, p_3} \int \frac{dp_4}{2\pi} \exp(i p_4 t) [B(p)]_{\mathbf{p}=0} \; .$$

It is best to calculate the integral over k_4 first, then p_4.

Show that when $C = 1$:

We will want to show for $C = 1$ that

$$B(0) > 0, \frac{dB}{dt} < 0 \text{ for all t} , \tag{1}$$

but that for $C = 1/4$ you can find at least one value of t where $B(t) < 0$.

Finally, evaluate this expression in the large t limit, where only the $\mathbf{k} = 0$ term in the sum over $\mathbf{k}$ contributes.

Show trivially that

$$B(\infty) = 0 , \tag{2}$$

for any value of C. With Eqns. 1 and 2, this proves that the a_0 correlator is guaranteed to be positive for unrooted staggered quarks but not for rooted.