Calculating the Flavor-Neutral Propagator

In this problem you will determine the inverse of the full flavor-neutral propagator in a more elegant way than we worked in the lecture.

1. First, show that the operator defined in the lectures

$$A = \frac{G_0 V}{\mathrm{tr}(G_0 V)}$$

is in fact a projection operator, that is, $A^2 = A$, when

$$(G_0)_{FF'} = \frac{\delta_{FF'}\epsilon_F}{p^2 + m_F^2}, \text{ no sum}, \qquad V_{IJ} = \frac{m_0^2}{3}\epsilon_I \epsilon_J \ ,$$

where we are using capital letters to denote the flavor-neutral mesons, and the ϵ is defined by the quark which makes up a given flavor.

2. The full propagator is given by

$$G = (G_0^{-1} + V)^{-1} = (1 + G_0 V)^{-1} G_0 \ .$$

Using that for a projection operator A and for any function f

$$f(A) - f(0) = A[f(1) - f(0)] \ ,$$

show that

$$(1 + G_0 V)^{-1} = 1 - \frac{G_0 V}{1 - \mathrm{tr}(G_0 V)}$$

3. Show that

$$\frac{\det(G^{-1})}{\det(G_0^{-1})} = 1 + \mathrm{tr}(G_0 V) = 1 + \mathrm{tr}(\bar{G}_0 \bar{V})$$

For the first equality, it is useful to combine the determinants into a single one, and insert the expression for G^{-1} in terms of A, the projection operator defined above. The bar over the matrix G_0 or V means that we are restricting the matrix to only include the sea sector, so the second equality can be shown explicitly by evaluating $\mathrm{tr}(G_0 V)$. The is important because it implies that the only contributions to the determinant come from the sea mesons, and all valence/ghost contributions vanish.

4. Putting all of this together, you can now show that

$$G_{FF'} = \frac{\delta_{FF'}\epsilon_F}{p^2 + m_F^2} - \frac{m_0^2}{3} \frac{(p^2 + m_U^2)(p^2 + m_D^2)(p^2 + m_S^2)}{(p^2 + m_F^2)(p^2 + m_{F'}^2)(p^2 + m_{\pi^0}^2)(p^2 + m_\eta^2)(p^2 + m_{\eta'}^2)}$$

in the case of an $SU(6|3)$ partially quenched theory.