# Positivity of the Flavor-Neutral Propagator

In the limit $m_0 \to \infty$, we have for the flavor-neutral propagator

$$G_{FF'} = \frac{\delta_{FF'}}{p^2 + m_F^2} - \frac{1}{3} \frac{(p^2 + m_U^2)(p^2 + m_D^2)(p^2 + m_S^2)}{(p^2 + m_F^2)(p^2 + m_{F'}^2)(p^2 + m_{\pi^0}^2)(p^2 + m_\eta^2)}$$

where $F, F'$ are valence mesons. If we take the isospin limit, then we have the eigenvalues of the full flavor-neutral mass-squared matrix

$$m_{\pi^0}^2 = m_U^2 = m_D^2 \ , \quad m_\eta^2 = \frac{1}{3}(m_U^2 + 2m_S^2)$$

with eigenvectors

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}(|U\rangle - |D\rangle) \ , \quad |\eta\rangle = \frac{1}{\sqrt{6}}(|U\rangle + |D\rangle - 2|S\rangle) \ .$$

The propagators then can be written

$$G_{\pi^0\pi^0} = \frac{1}{2}(G_{UU} + G_{DD} - 2G_{UD}) \ ,$$

$$G_{\eta\eta} = \frac{1}{2}(G_{UU} + G_{DD} + 4G_{SS} + 2G_{UD} - 4G_{US} - 4G_{DS}) \ ,$$

and

$$G_{\pi^0\eta} = 0 \ .$$

Show that in the limit of full QCD, that these propagators have no double poles and are positive definite, and have the form you would expect, starting from

$$G_{FF'} = \frac{\delta_{FF'}}{p^2 + m_F^2} - \frac{1}{3} \frac{(p^2 + m_U^2)(p^2 + m_S^2)}{(p^2 + m_F^2)(p^2 + m_{F'}^2)(p^2 + m_\eta^2)} \ .$$

Note this is non-trivial. We are showing explicitly that the violations of unitarity which clearly appear in the partially quenched theory can be removed by taking the full QCD limit. In other words, it is showing explicitly that full QCD is a limiting case of this particular $SU(6|3)$ partially quenched theory.